FACTORS AFFECTING COMPREHENSION OF MATH WORD PROBLEMS—A REVIEW OF THE RESEARCH

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The research that focuses on students' comprehension of math word problems can be viewed from the math eduator's perspective, the reading educator's perspective, or from a pedagogical perspective (Cohen, 1981).

The math educator's perspective suggests that to improve a student's comprehension of math word problems, the instructor must concentrate on the teaching of math concepts, procedures, generalizations, logical exchanges, and number facts. In order to read and work math problems successfully, one must understand the logical exchanges or moves (Davis, 1978) involved in solving the quantitative problems.

The reading educator's perspective would relate difficulty with word problems in math to linguistic and psycholinguistic comprehension theories. One such theory is Carver's (1977-78) rauding model which uses two variables that deal with written discourse and two that represent reader characteristics to predict comprehension performance. Pearson and Johnson (1978) would criticize Carver's model as too simple and would use more elaborae theories of comprehension to describe comprehension of word problems in math. "Whether the theory approaches the parsimony of Carver's or the complexity of Pearson's, these reading educators perceive comprehension as an interaction of text characteristics with reader characteristics" (Cohen, 1981, p. 177). The math educator's perspective puts most of the emphasis on the characteristics of the learner. This perspective tends to concentrate on strategies of instruction and the nature of problem solving, rather than focusing on written discourse and the subject's reading aptitude.

The pedagogical perspective would view the understanding of word problems in math in terms of a behavioristic approach such as that offered by Skinner (1969). This perspective would focus on the possibility of directly modifying behavior of math students rather than testing theories. This approach would begin with data in search of a theory. Both the math and reading educator's perspectives begin with a theory in search of data.

The author will attempt a review of the literature for factors affecting the reading of mathematics. This review will emanate from these three perspectives.

The Math Educator's Perspective

The perspective of the math educators places emphasis on the math aptitude of the learners. This view would suggest that students must possess certain understandings to succeed in mathematics. Piaget (1953) postulated that one such prerequisite for young subjects was conservation. He contended that a student does not acquire mathematical relationships through verbalizations, but only when his mental maturity is sufficient to grasp the principle of conservation of quantity. The student can then see that the number of items in a group remains the same regardless of how they are arranged. "Any adult attempt to impose mathematical concepts on a child prematurely results in verbal learning only" (Corle, 1972, p. 76).

Piaget also introduced the concept called reversibility. This prerequisite necessitates an understanding of the fact that regardless of how the quantity is manipulated, it can be restored to its original state by an inverse action (e.g., $7-4=3, 3+4=7$). Acording to Piaget, if a student lacks the capability to perceive such relationships, he cannot succeed in mathematics.

Brace and Nelson (1966) found a positive correlation between five- and six-year-old children's knowledge of cardinal number and their conservation abilities. This relationship decreased with age. Murray (1965) stated that the transition from nonconservation to conservation occurs sometime between the ages of seven and eight. "Any training procedures prior to this transition age appeared only to result in memorization of statements about the abstractions rather than to provide an awareness of the meanings of the relationships themselves" (Corle, 1972, p. 77). Almy (1966) investigated five- to eight-year-olds to determine the age at which students attained reversibility. Almy's study showed that the age of reversibility for most of the middle-class subjects was seven years and four months. Her study supported Piaget concerning the maturational development of a student's logical abilities.

So, once again the age-old question appears concerning the link between maturation and environment. Most researchers, however, are cognizant of the folly inherent in a determination to account for all the variance in math achievement by such simplicity. Intellectual development while certainly related to achievement in about anything, has always accounted for only a portion of the variance when the criterion was reading achievement, math achievement, or delivery of a paper at the American Reading Forum.

Hoel (1954) concluded from a study of math underachievers that emotional difficulty was responsible for most of the failure. Capps (1962) concluded from a study of accelerated and retarded students in math that underachievement in math was related to personal adjustment. Examination of the mean intelligence quotient for retarded math groups indicated that these fourth and six grade subjects had average mental ability. Bruekner and Grossnickle (1953) specified the inter-correlations between intelligence and various math skills, with a low of .35 between I.Q. and math computation skills.

But what about the correlation between math computation skills and higher problem-solving ability? The math educator's perspective suggests that the correlation between math computation skills and the reading of math word problems should be positive and high. Balow (1964) studied 1400 sixth-grade subjects to determine the importance of reading ability and computational ability in solving math word problems. Balow discovered that general reading ability and computation ability had a significant effect on problem-solving ability. Scores on the math word problem test showed a closer relationship to computation ability than to reading ability.

Stern and Keislar (1967) investigated the utility of offering instruction in problem-solving strategy to third graders. Their study demonstrated that children who were taught strategies for solving math word problems were significantly more successful than children who had not been given such instruction. Cathcart and Liedtke (1969) studied the importance of selecting the proper solution process for a math word problem. Their findings indicated that reflective students who reflected upon the quality of their answers achieved better scores than the impulsive subjects who gave unconsidered responses.

The math educator's perspective with a focus on the learner and strategies for math instruction does have support in the research literature. Maturation and intelligence are related to math learning. The writer wishes to offer a syllogism. Organisms tend to learn what they are taught. Organisms that are taught strategies for accomplishing something are much more successful in applying those strategies than organisms that are not taught. Subjects are organisms. Subjects that are taught strategies for solving math word problems are more successful in applying these strategies than subjects who are not taught.

The Reading Educator's Perspective

It seems less than sagacious to suggest that numerical understanding alone will not guarantee success in mathematics, especially in math word problems.

Vanderline (1964) and Lyda and Duncan (1967) collected data which demonstrated that the direct study of math vocabulary alone produced a significant growth in elementary students' problem-solving abilities. Chase (1961) concluded that the ability to note details in math word problems was a skill necessary for success.

The textbook in many school systems is the only resource provided for instruction. Reys and Knowles (1968) surveyed elementary schools and found that two-thirds of the schools used only one textbook for math instruction. This dependence upon textbooks underscores the significance of the reading educator's perspective.

Repp (1960) counted 3,329 words taken from five third-grade texts and found that approximately 1500 to 2000 were new to third graders. Reed (1965) found no significant agreement between the mathematics vocabulary of the textbook and the reading series used by her students. She found no significant agreement between the mathematics vocabulary of the textbook and standard word lists.

"There is considerable evidence that vocabulary specialists have discovered a disproportionate number of unfamiliar words in the mathematics books used by young children" (Corle, 1972, p. 86). Heddens and Smith (1964) used the Spache readability formula for grades one thru three, and the Dale-Chall formula for grades four thru six, to study five commercial math texts. These researchers concluded that all five series showed readability levels above the assigned grade levels.

Faison (1951) studied 38 texts from grades five thru eight and compared both the level of difficulty and the interest potential of the texts. He concluded that the math books were the hardest to read and ranked next to the lowest in interest. Faison's study was recently supported by an interesting piece of research by Elliott and Wiles (1980). They investigated the difficulty of current math textbooks by administering a cloze test from an eighth grade mathematics book to 91 certified mathematics teachers. Approximately 27% of the 91 teachers received scores of 55% or less correct.

Some studies have been conducted which dealt with both mathematical capabilities and reading skills. Chase (1960) studied 15 variables that might affect an intermediate subject's ability to solve math word problems and concluded that the ability to compute, skill in noting details in reading, and a knowledge of arithmetic concepts were the best three predictors of problem solving efficiency. Glennan and Callahan (1968) concluded that the most important factors were: general reading skills such as vocabulary knowledge, comprehension of the problem statement, and selection of relevant details; mechnical computation with a mathematical understanding of the concept of quantity, the number system, and arithmetic relationships; and a spatial factor involving the ability to visualize objects and symbols in more than one dimension.

While the math educators' perspective has support in the research literature, certain reading skills are also important for success in solving math word problems. Vocabulary development and literal interpretation of the problem seem crucial. Textbook readability is a major factor in relation to these skills.

The Pedagogical Perspective

The experiments in this section derive from the pedagogical perspective and recognizes the Skinnerian argument that theory building should start from data collected as a result of modifying human behavior.

An initial study that began the task of isolating format variables which interfere in math word problems was conducted by Loftus and Suppes (1972). They identified and defined eight such variables. Included was the order in which the math word problem was stated as compared to the order necessary to perform the appropriate computation. These researchers also listed the number of words in the problem as an important variable. Searle, Lorton, and Suppes (1974) used a step down regression model and identified how much each of 23 format variables contributed to the difficulty of math word problems.

Cohen (1981) continued with this research. He had students rewrite typical math textbook word problems to make them more understandable for other students having trouble with math. The purpose of the Cohen study was to discover the variables students perceived as difficult, and to compare these with the variables Searle et al. (1974) identified with the step down regression model.

Cohen chose 35 gifted sixth- and eighth-grade students with an I.Q. of 132 and above to rewrite 15 math word problems. One-half of the subjects had scored only average in math achievement as measured by grade level CTBS scores. The subjects rewrote 15 math word problems within a single class period in an attempt to make the problems easier to understand.

Cohen's content analysis of the students' rewritten items found 12 different format changes. He chose three of the variables in the format changes that concurred with variables reported by Searle et al. (1974) for further study as to their potency in solving word problems. Cohen chose variables that were amenable to student manipulation and for which it would be simple to construct instructional materials. The variables identified as interfering with math word problems were the absence of a diagram, the presence of extraneous information, and incorrect order of numbers in a word problem so that the presentation of the numbers appeared in an order other than that required for the appropriate solution.

Cohen then presented 225 average sixth graders with a 15 item test which included easy and difficult items in relation to the variables. The presence of diagrams in the math word problems, the reordering of the sequence in which numbers were presented in the problems to conform to the order required for appropriate solution, and the elmination of extraneous information all led to a significant increase in student performance in solving math word problems. The students were able to execute the arithmetic and logic of the word problems when presented in the easier formats, but encountered significant difficulty when attempting the more difficult formats. Computation was not a problem for these students. Format, however, affected comprehension.

Cohen conducted a third experiment (1981) to determine if subjects could be trained to insert a diagram when there was none, to extract extraneous information, and to reorder number sequence when it was inappropriate. The purpose of the third experiment by Cohen was to estimate how large the direct instruction effect would be after three class hours of instruction.

Seventy-one average sixth-grade students were placed in three treatment groups which received instruction in one treatment. The treatment groups used programmed materials which explained the task, demonstrated it, and then provided practice with immediate feedback. Each subject took a 21-item post test.

The findings demonstrated that when students were taught to insert diagrams in word problems that lend themselves to diagrams, they were significantly more successful than the control group. The instructional effect accounted for over 60% of the total variance. Instruction in extracting extraneous information accounted for over 65% of the total variance. Instruction in ordering numerical information accounted for 37% of the total variance.

Cohen's research is supported by Burns and Yonally (1964) who, using fourth and fifth graders, tried different ways of ordering the numerical information in math word problems. They found that subjects were less successful in getting correct answers when the numerical information was not in the order needed.

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